Colorado Teacher-Authored Instructional Unit Sample

Mathematics
6th Grade

Unit Title: Go Figure!

Colorado's District Sample Curriculum Project

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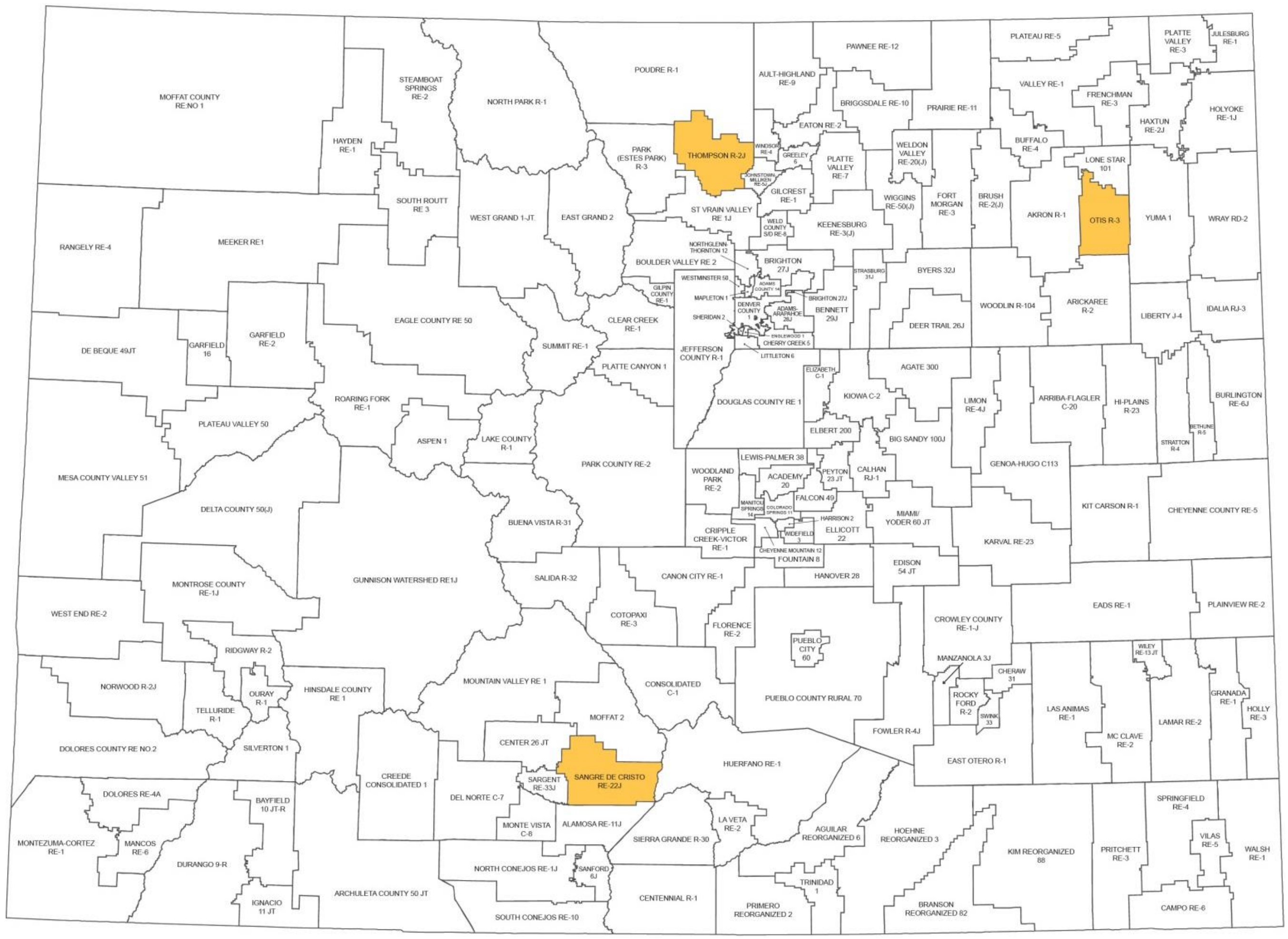


This unit was authored by a team of Colorado educators. The template provided one example of unit design that enabled teacher-authors to organize possible learning experiences, resources, differentiation, and assessments. The unit is intended to support teachers, schools, and districts as they make their own local decisions around the best instructional plans and practices for all students.

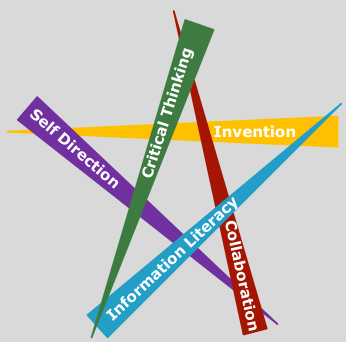
DATE POSTED: MARCH 31, 2014

Colorado Teacher-Authored Sample Instructional Unit

Content Area	Mathematics	Grade Level	6th Grade
Course Name/Course Code			

Standard	Grade Level Expectations (GLE)	GLE Code
1. Number Sense, Properties, and Operations	1. Quantities can be expressed and compared using ratios and rates	MA10-GR.6-S.1-GLE.1
	2. Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency	MA10-GR.6-S.1-GLE.2
	3. In the real number system, rational numbers have a unique location on the number line and in space	MA10-GR.6-S.1-GLE.3
2. Patterns, Functions, and Algebraic Structures	1. Algebraic expressions can be used to generalize properties of arithmetic	MA10-GR.6-S.2-GLE.1
	2. Variables are used to represent unknown quantities within equations and inequalities	MA10-GR.6-S.2-GLE.2
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge	MA10-GR.6-S.3-GLE.1
4. Shape, Dimension, and Geometric Relationships	1. Objects in space and their parts and attributes can be measured and analyzed	MA10-GR.6-S.4-GLE.1

Colorado 21st Century Skills



Critical Thinking and Reasoning: *Thinking Deeply, Thinking Differently*

Information Literacy: *Untangling the Web*

Collaboration: *Working Together, Learning Together*

Self-Direction: *Own Your Learning*

Invention: *Creating Solutions*

Mathematical Practices:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Titles	Length of Unit/Contact Hours	Unit Number/Sequence
Go Figure!	4 weeks	5

Unit Title	Go Figure!		**Length of Unit**	4 weeks
Focusing Lens(es)	Application Tools	**Standards and Grade Level Expectations Addressed in this Unit**	MA10-GR.6-S.1-GLE.3 MA10-GR.6-S.4-GLE.1	
Inquiry Questions (Engaging-Debatable):	• Can a triangle and a parallelogram have the same area? (MA10-GR.6-S.4-GLE.1-EO.a)			
Unit Strands	The Number System, Expressions and Equations, Geometry			
Concepts	Volume, three-dimensional figures, surface area, composing, decomposing, shapes, area, formula, x- and y- coordinates, coordinate plane, horizontal, vertical, line, graphing, distance, difference, fractional part, unit cube, equipartitioning, dimensions (length, width, height)			

Generalizations **My students will Understand that…**	**Guiding Questions** **Factual**	**Conceptual**
Decomposing three-dimensional figures into nets facilitates calculation of surface area. (MA10-GR.6-S.4-GLE.1-EO.d.i, d.ii, d.iii)	What is a net? What is an edge? Face? Vertices?	How does a net represent surface area? Why can two figures with the same surface area have the same volumes and vice versa?
Composing and decomposing shapes maintains the attribute of area. (MA10-GR.6-S.4-GLE.1-EO.a.i, a.ii)	How can every parallelogram be decomposed and rearranged into a rectangle with the same height and base?	Why does decomposing a shape maintain area? Why is every triangle half of a parallelogram? Why is the formula for the area of a rectangle the basis for most area formulas?
On a coordinate plane, points with the same x-coordinate form a vertical line when connected, and points with the same y-coordinate form a horizontal line when connected. (MA10-GR.6-S.4-GLE.1-EO.c.ii)	How can you find the distance between two points that form a vertical or horizontal line on a coordinate plane? Without graphing, how can you tell if two points will form a vertical or horizontal line?	Why do points with one matching coordinate form horizontal or vertical lines when connected? Why is the length of non-horizontal or vertical line on a coordinate grid determined by the number of squares it passes through?
The equipartitions of a unit cube into smaller components using a composition of splits along each dimension provides a visual representation of the value of fractional amounts (length, width, and height). (MA10-GR.6-S.4-GLE.1-EO.b.i, b.ii)	What is the size of a right rectangular prism with side lengths of 1/5, 1/2, and 1/4 units? What is the formula of right rectangular prism volume?	How does packing a right rectangular prism with fractional edge lengths by unit cubes of the appropriate unit fraction edge lengths help show that the volume of the prism is the same as would be found by multiplying the edge lengths?

Key Knowledge and Skills: **My students will…**	*What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.*

- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane and include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinates. (MA10-GR.6-S.1-GLE.3-EO.d)
- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (MA10-GR.6-S.4-GLE.1-EO.a.i, a.ii)
- Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism and apply the formulas $V = l\ w\ h$ and $V = B\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (MA10-GR.6-S.4-GLE.1-EO.b.i, b.ii, b.ii)
- Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate and apply these techniques in the context of solving real-world and mathematical problems. (MA10-GR.6-S.4-GLE.1-EO.c.i, c.ii)
- Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures and apply these techniques in the context of solving real-world and mathematical problems. (MA10-GR.6-S.4-GLE.1-EO.d.i, d.ii, d.iii)

Critical Language: includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline.

EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: *"Mark Twain exposes the hypocrisy of slavery through the use of satire."*

A student in ______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):	*When graphing a polygon on the coordinate plane using ordered pairs, I can calculate the distance of the side lengths if the either the x or y coordinates are the same for the two vertices of a side.*
Academic Vocabulary:	composing and decomposing, triangle, rectangle, horizontal, vertical, graph, distance, dimensions, length, width, height, draw,
Technical Vocabulary:	vertices, edge, face, absolute value, net, polygon, parallelogram, quadrilaterals, rectangular prism, three-dimensional, volume, area, surface area, formula, coordinate plane, quadrant, ordered pairs, coordinates, difference, fractional part, unit cube

Unit Description:	This unit focuses on the geometric concepts of perimeter, area, surface area and volume. Students begin by working on the coordinate plane to find the perimeter and area of rectangles. The formula for the area of a rectangle is then built upon to create a formula for the area of parallelograms and triangles. The concept of area is then extended to surface area and nets of three-dimensional solids with rectangular and triangular bases. Students finish the unit by exploring volumes of right rectangular prisms with fractional side lengths by connecting back to area and fraction multiplication.
Unit Generalizations	
Key Generalization:	Decomposing three-dimensional figures into nets facilitates calculation of surface area
Supporting Generalizations:	Composing and decomposing shapes maintains the attribute of area
	On a coordinate plane, points with the same x-coordinate form a vertical line when connected, and points with the same y-coordinate form a horizontal line when connected
	The equipartitions of a unit cube into smaller components using a composition of splits along each dimension provides a visual representation of the value of fractional amounts (length, width, and height)

Performance Assessment: *The capstone/summative assessment for this unit.*	
Claims: (Key generalization(s) to be mastered and demonstrated through the capstone assessment.)	Decomposing three-dimensional figures into nets facilitates calculation of surface area.
Stimulus Material: (Engaging scenario that includes role, audience, goal/outcome and explicitly connects the key generalization)	You work for a candy company that has just developed a new type of candy. The candy is 1 cm deep and 2 cm across. The owner of the company asked several candy box makers to create packaging for the new candy. The candy box makers were provided the following specifications: • design a box that could hold 18 pieces of candy (i.e., net) • minimize the packaging material (i.e., surface area) • create an appealing design Several companies have submitted designs. You need to write an evaluation of each design based on the above specifications to help the owner choose the best packaging company.
Product/Evidence: (Expected product from students)	Students will write an evaluation for each of the three possible candy packages and provide a recommendation to the owner. When evaluating each candy box students need to determine if: • net would create a box • surface area of the box is smaller than the other boxes • volume of the box would allow for 18 pieces of candy • box has an appealing design • design could be improved • design should be chosen by the owner This performance task is based on a lesson from the Shell Center. Resources for the lesson such as example designs of the candy boxes for students to evaluate can be found at: http://map.mathshell.org/materials/download.php?fileid=1364

Differentiation: (Multiple modes for student expression)	Students can work with partners on exploring the candy box designs before writing their own evaluations. Students can use an evaluation form with questions to scaffold their evaluation: http://map.mathshell.org/materials/download.php?fileid=1364 Students can create two of their own candy box designs, compare their designs and explain which of their designs is the best; rather than evaluating the designs of other people.

Texts for independent reading or for class read aloud to support the content

Informational/Non-Fiction	Fiction
	Perimeter, Area, and Volume, by David A. Adler (Lexile level 740) *A Fly on the Ceiling* by Julie Glass (Lexile level 560) *Mapping Shipwrecks With Coordinate Planes* by Julia Wall (Lexile level 760)

Ongoing Discipline-Specific Learning Experiences

1.	Description:	Think/work like a mathematician – Expressing mathematical reasoning by constructing viable arguments, critiquing the reasoning of others [Mathematical Practice 3]	Teacher Resources:	http://schools.nyc.gov/Academics/CommonCoreLibrary/TasksUnitsStudentWork/default.htm (lesson plans contains exemplars that could be replicated for students to critique the reasoning of others)
			Student Resources:	N/A
	Skills:	Construct and communicate a complete and concise response, justify a conclusion using correct vocabulary, interpret and critique the validity of other's conclusions and reasoning, and identify errors and present correct solutions	Assessment:	Students analyze and defend their solutions for each major learning experience. Careful attention should be paid to precise use of vocabulary and symbols. Periodically throughout the unit, students could be provided with flawed solutions and asked to identify, describe, and correct the flaw.
2.	Description:	Think/work like a mathematician – Engaging in the practice of modeling the solution to real world problems [Mathematical Practice 4]	Teacher Resources:	https://www.sites.google.com/a/cmpso.org/caccss-resources/k-8-modeling-task-force/k-8-modeling-resources (examples of modeling problems and resources for teachers on teaching and scoring them) http://www.insidemathematics.org/index.php/standard-4 (video examples of students modeling with mathematics) http://learnzillion.com/lessons/1722-solve-multistep-word-problems-using-model-drawing (video about modeling)
			Student Resources:	N/A

	Skills:	Devise a plan to apply mathematics to solve everyday problems by using and making stated assumptions and approximations to simplify a real-world situation. Map relationships between important quantities by selecting appropriate tools to create models. Analyze mathematical relationships between important quantities to draw conclusions. Determine if the results make sense. If necessary, change the model.	Assessment:	Modeling Problems Students use geometric models to represent and analyze relationships of real world problems to draw conclusions and interpret results in relation to the context of the problem.
3.	Description:	Mathematicians fluently add, subtract, multiply, and divide multi-digit decimals	Teacher Resources:	http://melrose.patch.com/groups/margaret-adamss-blog/p/bp--activities-at-home-for-increasing-math-fluency (recommendations to support fluency) http://www.mathwire.com/numbersense/bfacts.html (math games for the classroom to build fluency with basic facts)
			Student Resources:	http://www.coolmath-games.com/decimals-cruncher/ (fluency building game for decimals) http://www.memory-improvement-tips.com/math-mountain.html (fluency building game for all operations with whole numbers)
	Skills:	Add, subtract, multiply, and divide multi-digit decimals	Assessment:	Fluency Problems Students build fluency operating on multi-digit decimals by consistent practice with basic facts, multi-digit numbers and multi-digit decimals.

Prior Knowledge and Experiences

Student familiarity with the coordinate plane in the first quadrant will provide a strong foundation for the start of the unit. The concepts of perimeter, area, and volume for rectangles are an important building block throughout the unit, particularly the concepts of measuring area in square units and volume in cubic units.

Learning Experience # 1		
The teacher may provide points from all four quadrants and grid paper with coordinate planes so that students can relate the coordinates of a point to the movement of left, right, up and down. *Enactive*: Students can come to the front of the room in groups of three or four to "line dance". Each student can begin by standing at an origin and move to a point called out by the teacher by first moving the distance of the x-coordinate and then the distance of the y-coordinate, tile floors make this activity easier. Students can then move back to their origin by traveling the distance of x-coordinate and then the distance of the y-coordinate. Students can then repeat this process several times with points from all four quadrants. Teachers may then choose new students to line dance. *Iconic*: Students can trace the path (i.e., steps) of the students that are line dancing on a paper version of a coordinate plane. The path of the line dance will create a rectangle with one vertex at the origin and the opposite vertex at the point called out by the teacher. Students can label the point called out by the teacher. *Symbolic*: Students can plot points in all four quadrants on a coordinate plane without drawing the lines to trace their paths to the point.		
Teacher Notes:	This learning experience is a review of the coordinate plane including the concept of axes, origin and coordinates. These ideas were introduced in fifth grade and are extended to all four quadrants in this unit. It is important for students to stand at the intersection of the tiled floor and not in the middle of a tile. Students' prior experience with bar graphs or even the game battleship can often cause a misconception about the location of a point because students think the point is in the space of the grid rather than at the intersection.	
Generalization Connection(s):	On a coordinate plane, points with the same x-coordinate form a vertical line when connected, and points with the same y-coordinate form a horizontal line when connected	
Teacher Resources:	http://www.math-aids.com/Graphing/ (graphing worksheets) http://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-geometry-topic/cc-6th-coordinate-plane/v/the-coordinate-plane (coordinate plane video)	
Student Resources:	http://www.mathplayground.com/locate_aliens.html (find/plot the alien game) http://www.math-play.com/coordinate-plane-game.html (find the quadrant basketball game)	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: Without graphing, how can you tell if two points will form a vertical or horizontal line? Which axes do you move along as you plot an ordered pair? How can you tell the quadrant location of a point? What is another name for the horizontal axis? What is another name for the vertical axis? Why does every point have two coordinates?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may show the x and y-axis on the floor with tape	Students can "line dance" by beginning at an origin marked by a taped intersection Students can call out their movements. For example for the point (2, 4) students can call our right 1, 2, up 1, 2, 3, 4
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide points with fractional quantities for students to "line dance" and graph	Students can "line dance" and graph points with fractional quantities

Key Knowledge and Skills:	• Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane and include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinates
Critical Language:	Coordinate plane, axis, x-axis, y-xis, points, ordered pairs, quadrants, first quadrant, second quadrant, third quadrant, fourth quadrant, horizontal, vertical, polygons, quadrilaterals, vertices, grid line, intersection, origin

Learning Experience # 2

The teacher may explain the concept of absolute value as the distance between two points on a number line so that students can connect this idea to the coordinate plane and can graph the coordinates of a rectangle and find the side lengths and area of the rectangle.

Enactive: Students can work in groups of four to graph themselves on a life-size coordinate plane using points provided by the teacher. Students can then find the lengths of each side of the rectangle they created and explain how this relates to absolute value.

Iconic: Students can graph points on a coordinate plane (with grid lines) to create rectangles and find the perimeter and area of the rectangles.

Symbolic: Students can graph points on a coordinate plane (no grid lines) to create rectangles and find the perimeter and area of the rectangles. Students can then write an explanation of how to find the side lengths of a rectangle graphed on a coordinate plane (without grid lines) and its connection to absolute value.

Teacher Notes:	As students move to the symbolic stage and use coordinate planes without grid lines it may be helpful to keep the rectangles in only one of the four quadrants rather than crossing over the axes. Students may count the grid squares during the iconic stage. This can lead to a discussion about efficiency and the connection between multiplication and skip counting to find area. Finding the area of rectangles can be used as a formative assessment to determine if students need additional practice on the concept of area of rectangles before the learning experiences related to area of triangles and parallelograms.
Generalization Connection(s):	On a coordinate plane, points with the same x-coordinate form a vertical line when connected, and points with the same y-coordinate form a horizontal line when connected
Teacher Resources:	http://learnzillion.com/lessons/1066-find-perimeter-and-area-by-finding-the-length-of-sides-by-comparing-coordinates (video comparing coordinates to find distance) http://learnzillion.com/lessons/1147-use-absolute-value-to-find-distances-between-points (video about absolute value) http://learnzillion.com/lessons/677-determine-the-distance-between-integers-by-examining-absolute-value-and-number-lines (video about absolute value and distance between integers) http://www.graniteschools.org/depart/teachinglearning/curriculuminstruction/math/elementarymathematics/K6%20Support%20Documents/6th%20Grade%20Support/Absolute%20Value%20War%20-%20Game.pdf (teacher/student resource for a card game: Absolute Value War) http://www.opusmath.com/common-core-standards/6.g.3-draw-polygons-in-the-coordinate-plane-given-coordinates-for-the-vertices (teacher resource for making worksheets for counting and figuring area and perimeter) http://www.opusmath.com/common-core-standards/6.ns.8-solve-real-world-and-mathematical-problems-by-graphing-points-in-all-four?q=Find%20vertical%20and%20horizontal%20distances%20on%20the%20coordinate%20plane (teacher resource for making worksheets to find distances on a coordinate planes)
Student Resources:	http://www.xpmath.com/forums/arcade.php?do=play&gameid=96 (absolute value game on a number line)

<table>
<tr><td rowspan="2">**Assessment:**</td><td colspan="2">Students mastering the concept and skills of this lesson should be able to answer questions such as:
How can you find the distance between two points that form a vertical or horizontal line on a coordinate plane?
Without graphing, how can you tell if two points will form a vertical or horizontal line?
Why is distance related to absolute value?
Why is absolute value always positive?
Why is distance always positive?
Why do points with one matching coordinate form horizontal or vertical lines when connected?
Could you just count the number of squares a diagonal line passes through to determine its length? Why or why not?
How can you determine the area of rectangle in two different ways on a coordinate grid?
How can you find the fourth point of a rectangle if you are given three of the points?</td></tr>
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<table>
<tr><td rowspan="2">**Differentiation:**
(Multiple means for students to access content and multiple modes for student to express understanding.)</td><td>**Access** (Resources and/or Process)</td><td>**Expression** (Products and/or Performance)</td></tr>
<tr><td>The teacher may provide points only from the first quadrant
http://www.coolmath.com/prealgebra/08-signed-numbers-integers/04-signed-numbers-integers-absolute-values-01.htm (absolute value number line quick visual)</td><td>Students can create rectangles in the first quadrant and find the area and perimeter of each rectangle
Students can answer questions about absolute value by referring to visual descriptions of absolute value</td></tr>
<tr><td rowspan="2">**Extensions for depth and complexity:**</td><td>**Access** (Resources and/or Process)</td><td>**Expression** (Products and/or Performance)</td></tr>
<tr><td>The teacher may provide the coordinates for only two opposite vertices of a rectangle for a rectangle contained in all four quadrants</td><td>Students can graph rectangles when provided coordinates for only two of the vertices of the rectangle</td></tr>
<tr><td>**Key Knowledge and Skills:**</td><td colspan="2">• Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane and include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinates
• Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures and apply these techniques in the context of solving real-world and mathematical problems</td></tr>
<tr><td>**Critical Language:**</td><td colspan="2">Coordinate plane, axis/axes, x axis, y axis, points, ordered pairs, quadrant, horizontal, vertical, polygons, quadrilaterals, vertices, absolute value, grid lines, coordinates</td></tr>
</table>

Learning Experience # 3

The teacher may provide cardstock rectangles, straight edge, scissors, and tape so that students can explore the area of parallelograms in relation to a rectangle.

Enactive: Students can begin by finding the area of their rectangles. Students can then use a straightedge to draw a diagonal on their cardstock rectangle. Students can then cut along the diagonal to form two triangles. Students can then slide (e.g., translate) one of the triangles formed to the other side of the rectangle to form a parallelogram. Students can discuss with a partner how the original rectangle and the newly formed parallelogram compare to each other (e.g., area, base, and height stay the same). Students can then hypothesize how to find the area the parallelogram. Students can repeat the process of creating a parallelogram using their new parallelogram at least one more time (i.e., creating an even “skinnier” parallelogram with the same area).

Iconic: Students can draw a picture of their original rectangle and one of their parallelograms and label the base and height. Students can write about the comparisons they made with a partner between the rectangle and parallelogram and create a formula for finding the area of parallelogram based on the formula for the area of a rectangle.

Symbolic: Students can find the area of parallelograms using the formula A=bh.

Teacher Notes:	Students may find it helpful to have grid lines on the cardstock rectangles to reinforce the area of the rectangle compared to the area of the parallelogram. Often students learn the area of a triangle in relation to the area of a rectangle but this can lead to a limited view of triangles as only right triangles. By beginning with a parallelogram and then moving to triangles students can see all types of triangles are half the area of a parallelogram. A major goal of this learning experience is to help students see the difference between height and length. In a rectangle the height and length are the same but in a non-rectangular parallelogram the height and length are different.	
Generalization Connection(s):	Composing and decomposing shapes maintains the attribute of area	
Teacher Resources:	http://illuminations.nctm.org/Lesson.aspx?id=1882 (visual resource for cutting apart parallelogram) http://m.youtube.com/watch?v=hGGmu5D2bno (video about finding the area of a parallelogram by decomposing and composing rectangles) https://www.khanacademy.org/math/geometry/basic-geometry/area_non_standard/e/area_of_parallelograms (more instruction on area parallelogram)	
Student Resources:	http://www.ixl.com/math/grade-7/area-of-rectangles-and-parallelograms (area of rectangles and parallelograms)	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: What is the difference between height and length? How does the formula for the area of a rectangle compare to the formula for a parallelogram? How can a parallelogram be decomposed into a rectangle with the same height and base? Why use height and base rather than width and length? Does the formula for the area of a parallelogram work for a rectangle? Why or why not?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide two identical rectangles The teacher may provide a drawing of a rectangle and parallelogram	Students can cut apart one of the rectangles to make a parallelogram and use the second rectangle for comparing the height, base, side lengths and area Students can label a pre-drawn rectangle and parallelogram with base and height
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide a variety of parallelograms, scissors and tape	Students can develop the formula for the area of a parallelogram by exploring a variety of parallelograms and then compare their formula for the area of a parallelogram with the formula for the area of a rectangle
Key Knowledge and Skills:	• Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems	
Critical Language:	Area, parallelogram, height, base, slide (translation), rectangle, formula, width, length, compare, triangle, hypothesis, decompose, compose	

Learning Experience # 4		
The teacher may provide identical pairs of cardstock triangles created on grid paper so that students can explore the area of triangles in relation to a parallelogram. *Enactive*: Students can create parallelograms from identical pairs of cardstock triangles. Students can then explain how to find the area of one triangle based on the area of the parallelogram to a partner. *Iconic*: Students can draw a picture of an acute, obtuse, and right triangle and a corresponding parallelogram labeling the base and height for each triangle and parallelogram. Students can then develop a formula for finding the area of a triangle and describe how the formula compares to the formula for the area of parallelograms. *Symbolic*: Students can find the area of triangles using the formula A= ½ bh.		
Teacher Notes:	The teacher may want grid lines on the card stock triangles to allow students to easily compare the base and height when they create parallelograms. Students may want to count the squares of the grid to find the area of the triangle; this will lead to an estimate of the area rather than a precise area. Students should be encouraged to use this estimate as a check when they develop a strategy (i.e., formula) for finding the area of the triangle.	
Generalization Connection(s):	Composing and decomposing shapes maintains the attribute of area.	
Teacher Resources:	http://www.mathgoodies.com/lessons/vol1/area_triangle.html (explanations of the area of a triangle in relation to the area of a parallelogram and examples problems)	
Student Resources:	http://www.factmonster.com/math/knowledgebox/player.html?movie=sfw41510 (game to find the area of triangle baseball game) http://www.ixl.com/math/grade-5/area-of-triangles (questions to practice finding the area of triangle)	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: What is the difference between height and length? How does the formula for the area of a parallelogram compare to the formula for a triangle? How can a triangle be composed into a parallelogram with the same height and base? Why should triangles be described with their height and base and not the words width and length? Why does decomposing a shape maintain area? Why is every triangle half of a parallelogram?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide drawings for acute, obtuse and right triangles and corresponding parallelograms	Students can label pre-drawn triangles and parallelograms with base and height
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide students with a variety of polygons (e.g., hexagon, octagon, quadrilateral) http://www.mangahigh.com/en-us/math_games/shape/length_area_and_volume/area_of_a_triangle (game to find the missing base or height)	Students can explain how to decompose any figure into triangles and then show how to find the area of the figure based on the area of triangles Students can find unknown quantities when provided the area of a triangle

Key Knowledge and Skills:	• Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems
Critical Language:	Area, parallelogram, height, base, slide (translation), rectangle, formula, width, length, compare, decompose, compose, triangle, angle, obtuse, acute, right, dimensions, polygon

Learning Experience # 5	
The teacher may provide templates of squares and equilateral triangles with congruent side lengths (1 unit) and models of rectangular prism (cube), triangular prism, triangular pyramid, and rectangular pyramid so that students can begin to experiment forming nets that create three dimensional figures and find the surface area of each net. *Enactive*: Students can cut out squares and triangles and tape them together to create nets for each type of three-dimensional figure. *Iconic*: Students can draw a sketch of the net for each type of three-dimensional figure. Students can then write the area inside each square or triangle (e.g., 1 square unit, ½ square unit) and represent the surface area as an addition equation of each part of the net. *Symbolic*: Students can be provided with two-dimensional drawings of three-dimensional figures and asked to represent the net and calculate the surface area of each net.	
Teacher Notes:	Students may struggle to create a net which folds into a three-dimensional figure. By using tape to attach the shapes students can modify their design if necessary. Also, students may want to cut out the figures with tabs on each side to make joining two shapes together easier.
Generalization Connection(s):	Decomposing three-dimensional figures into nets facilitates calculation of surface area
Teacher Resources:	https://grade5aglcommoncoremath.wikispaces.hcpss.org/6.G.4 (vocabulary, misconceptions, essential questions and video related to surface area) https://www.youtube.com/watch?v=DZ9xn_7HBGs (surface area video) http://nrich.maths.org/public/leg.php?code=-259 (net activities) http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/ (interactive surface area visual) http://illuminations.nctm.org/Activity.aspx?id=3521 (shows the movement from nets to solids)
Student Resources:	http://www.mangahigh.com/en-au/maths_games/shape/3d_shapes/nets (practice with nets and two-dimensional drawings of three dimensional figures) http://www.learner.org/interactives/geometry/area_surface.html (description of surface area and practice surface area problems)
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: How do you find the surface area of rectangular prism? What is a net? How does a net represent surface area? What is an edge? What is a face? What is a vertex?

Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://www.mathsisfun.com/geometry/vertices-faces-edges.html (visual images and vocabulary for three-dimensional figures) http://www.senteacher.org/worksheet/12/NetsPolyhedra.html (net templates)	Students can describe their nets and three-dimensional figures to a partner using the scaffold of visual images for key vocabulary Students can create three-dimensional figures and determine surface areas from provided net templates
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://www.korthalsaltes.com/ (examples of nets for other types of figures)	Students can create nets for complex three-dimensional figures (e.g., octahedron, icosahedron) and find the surface area for each net
Key Knowledge and Skills:	• Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures and apply these techniques in the context of solving real-world and mathematical problems	
Critical Language:	Rectangle, square, triangle, length, area, width, net, surface area, three dimensional figure, triangular prism, rectangular prism, rectangular pyramid, triangular pyramid, cube, height, base, edge, vertices, unit cube	

Learning Experience # 6

The teacher may provide the dimensions of several different sized rectangular prisms (including whole numbers and positive rational numbers) so that students can begin to connect the concept of volume with the concept of area in relation to fractional dimensions.

Enactive: Students can create a net on grid paper for a rectangular prism with whole number lengths and a height of one and fill the base with cubes. Students can explain to a partner how the area of the base relates to the volume of a prism with a height of one and how to use this idea to find the volume for other heights. Students can then create a net on grid paper for a rectangular prism with fractional side lengths and a height of one and determine the area of the base using fractional multiplication.

Iconic: Students can explain the formula for the volume of a rectangular prism, V = (area of base) X height, through drawings and connections to multiplication and area.

Symbolic: Students can find the volume of rectangular prisms when provided the dimensions of the prism.

Teacher Notes:	The cubes students use to fill their nets should be the same size as the grid paper. Students can estimate the area of the fractional side lengths by counting squares and partial squares and use the estimate as a check to their fractional multiplication.
Generalization Connection(s):	The equipartitions of a unit cube into smaller components using a composition of splits along each dimension provides a visual representation of the value of fractional amounts (length, width, and height)
Teacher Resources:	http://www.onlinemathlearning.com/volume-of-a-cube.html (volume explanation) http://www.interactivestuff.org/sums4fun/3dboxes.html (interactive applet find volume) http://learnzillion.com/lessons/1062-find-the-volume-of-a-rectangular-prism-by-filling-it-with-unit-cubes (video of filling a fish tank with cubes) http://learnzillion.com/lessons/1064-find-the-volume-of-a-rectangular-prism-with-fractional-edge-lengths (video on fractional side lengths) http://www.learner.org/interactives/geometry/area_volume.html (volume of a rectangular prism) http://www.shodor.org/interactivate/search?ss=fractional+volume (volume worksheets)

<table>
<tr><td>Student Resources:</td><td colspan="2">http://www.scholarnet.co.nz/member/courses/smol/data/site/flash_apps/Measurement.php (surface area and volume questions)</td></tr>
<tr><td>Assessment:</td><td colspan="2">Students mastering the concept and skills of this lesson should be able to answer questions such as:
What is the formula for the volume of a right rectangular prism?
How does area relate to volume?
Why is volume measured in cubic units?
Why is the formula for the volume of a rectangular prism V= lwh and V= Bh?
Why is the volume of a cube with side lengths ½ unit, 1/8 cubic units and not ½ cubic units?</td></tr>
<tr><td rowspan="2">Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>The teacher may provide dimensions of rectangular prisms with at most two fractional side lengths and/or fractions involving only halves or quarters
http://www.youtube.com/watch?v=uJ_rQ1gIXMM&list=PLnIkFmW0ticMeDHCHOzQ8VbnJwFwzEETV (video about the volume of a rectangular prism)</td><td>Students can calculate volume of prisms with at most two fractional side lengths and/or fractions involving only halves or quarters

Students can explain how to calculate the volume of a prism after watching a video about volumes of rectangular prisms</td></tr>
<tr><td rowspan="2">Extensions for depth and complexity:</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>http://illuminations.nctm.org/Lesson.aspx?id=2009 (table to compare volume to surface area starting with 1 cube)</td><td>Students can explain to the class using tables and visuals the relationships between volume and surface area
Students can find the volume of triangular prisms</td></tr>
<tr><td>Key Knowledge and Skills:</td><td colspan="2">• Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism and apply the formulas $V = l\ w\ h$ and $V = B\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems</td></tr>
<tr><td>Critical Language:</td><td colspan="2">Width, length, net, rectangle, surface area, three dimensional figure rectangular prism, height, edge, base, vertices, unit cube, volume, fractional part, layer, packing, cubic units</td></tr>
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