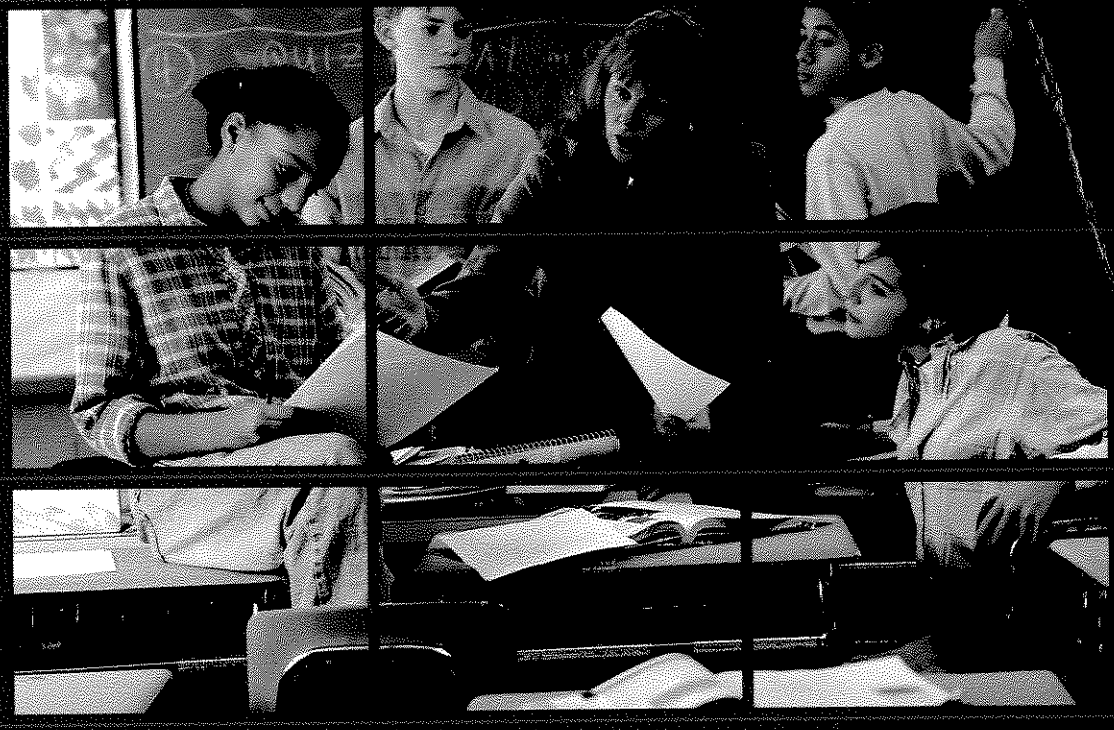
Balanced Assessment

MIDDLE GRADES ASSESSMENT



PACKAGE 2

Balanced Assessment for the Mathematics Curriculum

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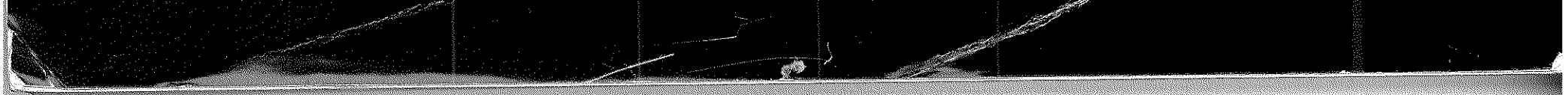


Expanded Table of Contents *

* For explanations of terms that may be unfamiliar, see the Glossary, and the *Dimensions of Balance* table in the Introduction.

Middle Grades Package 2

Mathematical Content	Mathematical Processes
Patterns, functions, and algebra: patterns; forming functional relationships; tabular and symbolic representation; properties of shapes	formulation of relationships through generalizing patterns; simplifying them by manipulation
Data and statistics: analysis of given data; rank ordering; grouping; evaluation over several variables	balance of interpretation and evaluation of the data; formulation of a model for grouping; evaluation and justification; communication of reasons
Geometry, space, and shape: location and symmetry, using properties of rectangles	some formulation of the approach and mainly of the consequent instructions; manipulation to work out the detail; communication
Patterns, functions, and algebra with number: measurement; generalizing; forming and solving relationships; symbolic representation	manipulation of the measurement, formulation of the relationship; communication of the results and the reasoning behind them
Geometry, space, and shape: locate points and determine slopes; angular properties of shapes; trigonometry; perpendicular and parallel lines	manipulation with some formulation
Data, statistics, and probability: probability concepts; combining and comparing probabilities; simulation of a game	formulation and manipulation of the probability model (theoretical or experimental); evaluation and communication of advice
Geometry, space, and shape: construct a system to locate points, lines, and circles on a grid to form a given pattern	formulation of the system; manipulation of the coordinates; communication in a clear message, with checks

Expanded Table of Contents

Short Tasks	Task Type	Circumstances of Performance
8. Grass for Goats	20-minute problem; pure mathematics; nonroutine context from student life	individual assessment
9. Picnic	20-minute recommendation and evaluation task; applied power; nonroutine context from student life	individual assessment
10. Polygon Measures	20-minute technical exercise	individual assessment
11. Library Books	15-minute technical exercise; applied power in a nonroutine context	individual assessment
12. Pat's Pattern	15-minute problem; pure mathematics; nonroutine result; open-middle	individual assessment
13. Secret Number	15-minute problem; pure mathematics; nonroutine math connections; open-ended	individual assessment
14. Vet Club	15-minute problem; applied power; nonroutine mathematical demands and student context	individual assessment
15. Framing Materials	10-minute problem; applied power; nonroutine adult-life context	individual assessment
16. Parallel Lines	15-minute problem; pure mathematics; nonroutine math connections	individual assessment
17. Tangram	10-minute problem; pure mathematics; nonroutine context	individual assessment
18. Tile Signs	15-minute problem; applied power; nonroutine context from adult life	individual assessment

Mathematical Content	Mathematical Processes
Geometry, space, and shape: area properties of circles; comparisons	mainly manipulation; straightforward application of the area formula; some formulation of approach
Number and quantity: concepts of and computation with decimals	mainly manipulation and evaluation of results; clear-minded formulation needed
Geometry, space, and shape: angle properties of regular polygons, their perimeters and areas; visualization	mainly manipulation
Data, statistics, and probability: bar graph interpretation; mean and median measures	interpretation of graphical data; manipulation; evaluation of results
Patterns, functions, and algebra: generalize a pattern sequence to form a functional relationship, linear in n	interpretation and analysis of the pattern sequence; formulation of the relationship; communication to a fellow student
Number and quantity: use theory of factors and primes; compute and evaluate possibilities	formulation of a strategy; manipulation to compute factors; evaluation of the clues
Data, statistics, and probability: compose a bar graph; choose mean or median measure of center; justify	interpretation of data; manipulation in forming a graph, and computation of measures; evaluation of results
Geometry, space, and shape, with number: properties of rectangles; compute complete set of possibilities	concept-based manipulation
Geometry, space, and shape: side and angle properties of similar and congruent triangles, formed by a grid of parallel lines	mainly manipulation, with some formulation of the approach
Geometry, space, and shape, with number: properties of triangles and rectangles; symmetry; fractions	formulation of a dissection of the square; manipulation to compute the fractions
Geometry, space, and shape, with number: properties of rectangles; compute complete set of possibilities	concept-based manipulation

Task 6

Overview

Reason about probability.
Justify a decision.
Communicate mathematical reasoning.

Lucky Draw

Long Task

Task Description

Students use theoretical and/or experimental probability to determine the profitability of a festival game. They write a report to the festival committee explaining their recommendation.

Assumed Mathematical Background

It is assumed that students have had experience with finding theoretical and experimental probability.

Core Elements of Performance

- analyze and reason about probability to determine the profitability of a carnival game
- make and justify a recommendation to an audience

Circumstances

Grouping:	Following a class introduction, students complete an individual written response.
Materials:	red and blue colored counters or cubes, 3 large cups (for class demonstration), and additional cups for students
Estimated time:	40 minutes

Name Date

Lucky Draw

This problem gives you the chance to

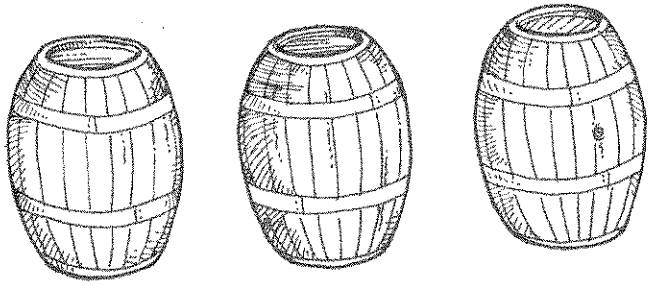
- *analyze and reason about probability*
- *make and justify a decision*
- *communicate mathematical reasoning*

At the Palatine School's fall festival, the Charity for Children Club is planning to run a money-raising booth. One of the members in the club proposed the following game:

Lucky Draw

Win $1.00

10¢ per turn



There are equal numbers of red and blue balls buried in sawdust in each barrel.

One turn allows you to make
ONE LUCKY DRAW from each barrel for 10¢.
If you draw three balls of the same color on one turn you win $1.00.

Elida, the chairperson of the festival, likes the idea of the game, but she wants to make sure it is a good money maker. As co-chairperson, you have been asked to prepare a report to the festival committee on this issue. Make sure that your report includes your recommendation and clearly explains how you came to your conclusion.

A Sample Solution

Task 6

Theoretical Approach A

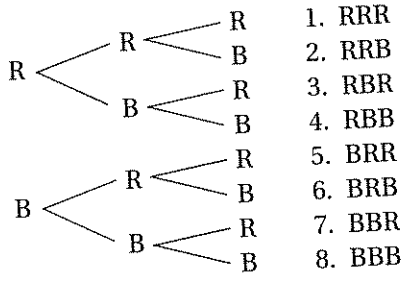
I believe that this game is not a good fund raiser. I figured out the following about the game:

- An equal number of red balls and blue balls means that there is a $\frac{1}{2}$ probability of drawing either red or blue from each barrel.
- If I want to draw three red balls (one from each barrel), then the probabilities must be multiplied: $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$. This is the same for three blue balls.
- So the probability of getting three balls of the same color is $\frac{1}{8}$ for red and $\frac{1}{8}$ for blue. So, together there is a $\frac{1}{4}$ probability of winning the \$1.00 prize.
- That means that for every four players, one of them should win. The game would take in \$0.40 for every four players and pay out \$1.00, leaving the charity \$0.60 behind. So they could expect to lose \$0.60 ÷ 4 players or \$0.15 per player on average when a lot of people play this game.

If a lot of people were to play this game, a lot of money could be lost and there would be no profit at all. I would not recommend this game to Elida as a "money maker."

Theoretical Approach B

To find out if the game would be profitable, I made a tree diagram. *R* is a red ball. *B* is a blue ball.



The diagram shows that there are eight different ways of drawing red and blue balls from the three barrels. Two of them, RRR and BBB, give the player a win. So there is a $\frac{2}{8}$ or $\frac{1}{4}$ chance of winning. So for every eight plays, the charity would take in \$0.80 and pay out \$2.00, which gives them a loss of

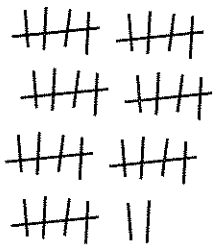
Task 6

$1.20. If a lot of people played this game, they would lose a lot of money. The game needs to be changed.

Experimental Approach

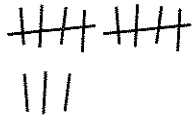
I played this game by putting an equal number of red and blue chips in three cups and then drew a chip from each cup 50 times (without looking). I put the chips back in their cups and mixed the chips very well after each play. Here are my results for the game:

Losses



37 losses
Charity club makes $3.70
(Each player paid $0.10,
so 37 × $0.10 = $3.70.)

Wins



13 wins
Charity club pays $11.70
(Each player paid $0.10, but the charity club paid out $1.00 to each of them,
so 13 × $1.00 = $13.00 – $1.30 = $11.70.)

Overall in this game, experimental probability shows that for every 50 players, we would make $3.70 on the losses and pay out $11.70 on the wins, giving a total loss of $8.00. I think I played this game enough times to show what will happen when a lot of people play it. I would expect that the charity will lose a lot of money with this game. It needs to be changed.

Using this Task

Task 6

For Formal Assessment

Hand out the task to students and read the aims of the assessment in the box at the top of the activity page. Read through the task with students and make sure they understand the context of the task. You may want to model the game with colored cubes or counters and cups. Have such materials available for students to use while completing the task. Answer any questions about the *context* of the task, but if students ask what form of probability they should use (experimental or theoretical), tell them it is their choice.

Issues for Classroom Use

Students may choose to use either theoretical or experimental probability to determine whether or not the game is profitable. If students use only experimental probability, a complete response must include an explanation of how the game was modeled (for example a student could draw three times from only one cup, replacing each ball after each draw) and must involve a large number of trials (50 or more). If students use *both* theoretical and experimental probability, a high-level response would include some discussion about the differences in the findings, the meanings of the two findings, and a reconciliation between the two in the recommendations to the festival committee.

This task may be extended in numerous ways. If many students use experimental probability, have them share the different findings and discuss why they may differ from each other and from the theoretical findings. Pose or have students pose different questions or change the parameters of the game (for example, what if there were four barrels to draw from, or n barrels? What if there were three different colors of balls? What if there were twice as many red balls as blue balls? What if we didn't know whether there were equal numbers of each color ball in each barrel?). Alternately, have students play with the different parameters—including cost to play and cost of prize—and have them design a profitable game.

Task 6

Characterizing Performance

This section offers a characterization of student responses and provides indications of the ways in which the students were successful or unsuccessful in engaging with and completing the task. The descriptions are keyed to the *Core Elements of Performance.* Our global descriptions of student work range from "The student needs significant instruction" to "The student's work meets the essential demands of the task." Samples of student work that exemplify these descriptions of performance are included below, accompanied by commentary on central aspects of each student's response. These sample responses are *representative;* they may not mirror the global description of performance in all respects, being weaker in some and stronger in others.

The characterization of student responses for this task is based on these *Core Elements of Performance:*

1. Analyze and reason about probability to determine the profitability of a carnival game.
2. Make and justify a recommendation to an audience.

Descriptions of Student Work

The student needs significant instruction.

Student shows some attempt at reasoning through the task, but does not show understanding of how to apply probability analysis to the situation.

Student A

Student A shows some attempt at reasoning through the task. He recommends charging a higher price (60¢) to play the game and correctly determines that with a 10¢ cost to play, "they would lose lots of moolah." His diagram (with the Bs and Gs) may represent some attempt at modeling the theoretical probability of winning the game, but his reasoning is unclear.

Task

6

The student needs some instruction.

Student uses probability to reason through the task, but the student's reasoning is based only on experimental probability with an insufficient number of trials (less than 50) *OR* student does not model theoretical outcomes completely and correctly.

Student B

Student B correctly determines that the game is not profitable based on her experimental findings, but she conducts only 15 trials, an insufficient number. Student B also models the game incorrectly. Although she placed equal amounts of red and blue balls in a bowl and closed her eyes, she appears to have drawn all three balls at once, without replacing them in between draws. This would have affected the probabilities of the different possible outcomes.

Student C

Student C's tree diagram correctly lists all of the possible theoretical outcomes. However, Student C does not base his reasoning on this model. He explains, "It's a one in ten chance [of winning], so they wouldn't make very much money." Student C's theoretical probability model indicates a one in four chance of winning. He seems to base his reasoning on his experimental findings, shown in his diagram. This being the case, he conducted too few trials (only 10).

The student's work needs to be revised.

Student correctly determines that the game is not profitable through correct use of either theoretical or experimental probability, but student does not produce a clear and complete report directed to the festival committee.

Student D

Student D correctly uses theoretical probability to model the possible outcomes, to determine the probability of winning the game: "p (all r or b) = $\frac{2}{8}$," and to determine that the game would lose money (actually he shows how a player would make money). However, Student D does not direct his comments to the festival committee. He first says that the game is not fair (perhaps thinking that in a fair game, players have an equally likely chance to win or lose). Then as if talking to a player, he explains how you have a pretty good chance of winning money. (Student D also makes an arithmetical error. He says, "You would make 50¢." He should say, "60¢.")

Task

The student's work meets the essential demands of the task.

Student correctly uses experimental and/or theoretical probability to determine profitability and recommends rejecting the game *AND* student writes a clear and complete report to the festival committee, justifying the decision. (This includes an explanation of a correct modeling of the game if experimental probability is used.)

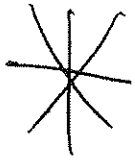
Student E

Student E correctly uses theoretical probability to determine the chances of winning the game and the profitability of the game. She writes a clear report to the committee, including sufficient justification of her recommendation. In addition, so that the game can produce a profit, Student E recommends a change in the cost to play and she explains the theoretical profit for the revised game.

Student A

Recommendation

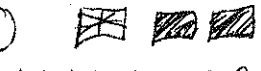
I think that they should charge 60¢ for each game, because its too easy to win right now and if they charged 10¢ they would lose lots of moolah



< 2 B-6
C-6
B-B

Student B

Lucky Draw

Yes NO 

1 1 1 1 = 4 $ 1111 1111 1 = 9 = 150

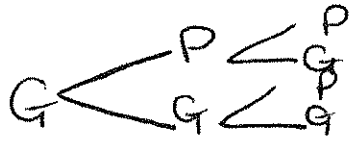
No it is not a good money-maker because you loose 2.50 on the winnings the customer gets. The Customer would have only spent 1.50 on 15 games and the workers would have to give back the Customer's money plus 2.50 more because that Customer won. I got my conclusion by putting 4 red balls and 4 blue balls into a bowl and I closed my eyes and picked out 3 balls.

Student C

Experimental:

Same	different
1	𝍸 𝍸 1111

Theoretical



Possible outcomes:

P-P-P
P-P-G
P-G-P
P-G-G
G-P-P
G-P-G
G-G-P
G-G-G

No, I don't think it's a good money maker because as soon as they get a dollar, someone wins so they don't get any money. It's a one in ten chance, so they wouldn't make very much money.

Student D

rrr	bbb
rrb	bbr
rbr	brb
rbb	brr

$P(\text{All } r \text{ or } b) = 0/8$

$P(\text{not all } r \text{ or } b) = 6/8$

It's not fair because you have a 1/4 chance of winning. You have a 3/4 chance of not winning. You have a pretty good chance of winning because theoreticaly if you draw 4 times then you'll win 1 time. If you, play 4 times thats only 40¢ to play, You would win 1 dollar and would make 50¢.

Student E

Notes, Research

Barrel #1	Barrel #2	Barrel #3
10¢ → R	R	R
10¢ → R	R	B
10¢ → R	B	R
10¢ → R	B	B
10¢ → B	B	B
10¢ → B	B	R
10¢ → B	R	B
10¢ → B	R	R

Out of eight draws you'd be expected to win twice. But if you only play once and win the booth will lose money.

Because in 8 turns you make 80¢ and lose $2.00, a lose at $1.20

Report

Elida is right, how it's set up now you prolly wouldn't make any or much money. My recommendation is to charge anywhere from 30¢ to 45¢ a turn. Because if you charge 35¢ and the person draws 8 times theoretically, you would make $2.80. If the person won twice, you would lose $2.00, profits theoretically for 8 turns is only 80¢. To come to this conclusion I first made an orginized list of all the possibilities. There were 8. Each person payed 10¢, 8 times 80¢, and winning twice is 2.00, 2.00 lose $1.20, Not good therefore raise your price or lessen your prize.