Colorado
Academic Standards

Mathematics

COLORADO
Department of Education

ALL STUDENTS • ALL STANDARDS

Mathematics Standards Review and Revision Committee

Chairperson

Joanie Funderburk
President
Colorado Council of Teachers of Mathematics

Members

Lisa Bejarano
Teacher
Aspen Valley High School
Academy District 20

Michael Brom
Assessment and Accountability Teacher on Special Assignment
Lewis-Palmer School District 38

Ann Conaway
Teacher
Palisade High School
Mesa County Valley School District 51

Dennis DeBay
Mathematics Education Faculty
University of Colorado Denver

Greg George
K-12 Mathematics Coordinator
St. Vrain Valley School District

Cassie Harrelson
Director of Professional Practice
Colorado Education Association

Lanny Hass
Principal
Thompson Valley High School
Thompson School District

Ken Jensen
Mathematics Instructional Coach
Aurora Public Schools

Lisa Rogers
Student Achievement Coordinator
Fountain-Fort Carson School District 8

David Sawtelle
K-12 Mathematics Specialist
Colorado Springs School District 11

T. Vail Shoultz-McCole
Early Childhood Program Director
Colorado Mesa University

Ann Summers
K-12 Mathematics and Intervention Specialist
Littleton Public Schools

State Board of Education and Colorado Department of Education

Colorado State Board of Education

Angelika Schroeder (D, Chair)
2nd Congressional District
Boulder

Joyce Rankin (R, Vice Chair)
3rd Congressional District
Carbondale

Steve Durham (R)
5th Congressional District
Colorado Springs

Valentina (Val) Flores (D)
1st Congressional District
Denver

Jane Goff (D)
7th Congressional District
Arvada

Rebecca McClellan (D)
6th Congressional District
Centennial

Debora Scheffel (R)
4th Congressional District
Parker

Colorado Department of Education

Katy Anthes, Ph.D.
Commissioner of Education
Secretary to the Board of Education

Melissa Colsman, Ph.D.
Associate Commissioner of Education
Student Learning Division

Floyd Cobb, Ph.D.
Executive Director
Teaching and Learning Unit

CDE Standards and Instructional Support Office

Karol Gates
Director

Carla Aguilar, Ph.D.
Music Content Specialist

Ariana Antonio
Standards Project Manager

Joanna Bruno, Ph.D.
Science Content Specialist

Lourdes (Lulu) Buck
World Languages Content Specialist

Donna Goodwin, Ph.D.
Visual Arts Content Specialist

Stephanie Hartman, Ph.D.
Social Studies Content Specialist

Judi Hofmeister
Dance Content Specialist
Drama and Theatre Arts Content Specialist

Jamie Hurley, Ph.D.
Comprehensive Health Content Specialist
Physical Education Content Specialist

Raymond Johnson
Mathematics Content Specialist

Christine Liebe
Computer Science Content Specialist

Vince Puzick
Reading, Writing, and Communicating Content Specialist

Purpose of Mathematics

"Pure mathematics is, in its way, the poetry of logical ideas."

~Albert Einstein, *Obituary for Emmy Noether* (1935)

"Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics."

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

1. To Expand Professional Opportunity. Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21st-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

2. Understand and Critique the World. A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

3. Experience Wonder, Joy, and Beauty. Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards:

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

1. Number and Quantity
From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

2. Algebra and Functions
Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

3. Data Analysis, Statistics, and Probability
From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

4. Geometry
Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

COLORADO
Department of Education

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

4.NBT.A. Number & Operations in Base Ten: Generalize place value understanding for multi-digit whole numbers.

Evidence Outcomes

Students Can:

1. Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that* $700 \div 70 = 10$ *by applying concepts of place value and division.* (CCSS: 4.NBT.A.1)
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. (CCSS: 4.NBT.A.2)
3. Use place value understanding to round multi-digit whole numbers to any place. (CCSS: 4.NBT.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Write multi-digit whole numbers in different forms to support claims and justify reasoning. (Entrepreneurial Skills: Literacy/Writing)
2. Use the structure of the base-ten number system to read, write, compare, and round multi-digit numbers. (MP7)

Inquiry Questions:

1. How do base ten area pieces or representations help with understanding multiplying by 10 or a multiple of 10? How can base ten area pieces be used to represent multiplying by 10 or a multiple of 10?
2. Imagine two four-digit numbers written on paper and some of the digits were smeared. If you saw just $325\blacksquare$ and $331\blacksquare$, could you determine which number was larger?
3. When is it helpful to use a rounded number instead of the exact number?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students use place value understanding and properties of operations to perform multi-digit arithmetic.
3. In Grade 4, this expectation connects to using the four operations with multi-digit whole numbers to solve measurement and other problems.
4. In Grade 5, students extend their understanding of place value to decimals, and read, write, and compare decimals to thousandths.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.NBT.B. Number & Operations in Base Ten: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Evidence Outcomes

Students Can:

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm. (CCSS: 4.NBT.B.4)
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.B.5)
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve multi-digit arithmetic problems. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Explain the process and result of multi-digit arithmetic. (MP3)
3. Precisely and efficiently add and subtract multi-digit numbers. (MP6)
4. Use the structure of place value to support the organization of mental and written multi-digit arithmetic strategies. (MP7)

Inquiry Questions:

1. How can a visual model be used to demonstrate the relationship between multiplication and division?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students use place value understanding and properties of operations to add and subtract within 1000 and to multiply and divide within 100.
3. In Grade 4, this expectation connects to using the four operations with whole numbers to solve problems.
4. In Grade 5, students understand the place value of decimals and perform operations with multi-digit whole numbers and with decimals to hundredths.

COLORADO
Department of Education

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.NF.A. Number & Operations—Fractions: Extend understanding of fraction equivalence and ordering.

Evidence Outcomes

Students Can:

1. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \times a}{n \times b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (CCSS: 4.NF.A.1)
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. (CCSS: 4.NF.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Explain the equivalence of fractions. (MP3)
2. Use visual models and benchmark fractions as tools to aid in fraction comparison. (MP5)
3. Precisely refer to numerators, denominators, parts, and wholes when explaining fraction equivalence and comparing fractions. (MP6)
4. Use 1, the multiplicative identity, to create equivalent fractions by structuring 1 in the fraction form $\frac{n}{n}$. (MP7)

Inquiry Questions:

1. Why does it work to compare fractions either by finding common numerators or by finding common denominators?
2. How can you be sure that multiplying a fraction by $\frac{n}{n}$ does not change the fraction's value?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students develop an understanding of fractions as numbers and the meaning of the denominator of a unit fraction.
3. In Grade 4, this expectation connects to building fractions from unit fractions, using decimal notation and comparing decimal fractions, and using the four operations with whole numbers to solve problems.
4. In Grade 5, students use equivalent fractions as a strategy to add and subtract fractions with unlike denominators and apply and extend previous understandings of multiplication and division to fractions.

Prepared Graduates:

MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

4.NF.B. Number & Operations—Fractions: Build fractions from unit fractions.

Evidence Outcomes

Students Can:

3. Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$. (CCSS: 4.NF.B.3)
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (CCSS: 4.NF.B.3.a)
   b. Decompose a fraction into a sum of fractions with like denominators in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$. (CCSS: 4.NF.B.3.b)
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (CCSS: 4.NF.B.3.c)
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. (CCSS: 4.NF.B.3.d)
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (CCSS: 4.NF.B.4)
   a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent* $\frac{5}{4}$ *as the product* $5 \times \frac{1}{4}$*, recording the conclusion by the equation* $\frac{5}{4} = 5 \times \frac{1}{4}$. (CCSS: 4.NF.B.4.a)
   b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express* $3 \times \frac{2}{5}$ *as* $6 \times \frac{1}{5}$*, recognizing this product as* $\frac{6}{5}$*. (In general,* $n \times \frac{a}{b} = \frac{n \times a}{b}$*.)* (CCSS: 4.NF.B.4.b)
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat* $\frac{3}{8}$ *of a pound of roast beef, and there will be* 5 *people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (CCSS: 4.NF.B.4.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use the structure of fractions to perform operations with fractions and to understand and explain how the operations connect to the structure of fractions. (MP7)
2. Recognize the mathematical connections between the indicated operations with fractions and the corresponding operations with whole numbers. (MP8)

Inquiry Questions:

1. How is the addition of unit fractions similar to counting whole numbers?
2. How does multiplying two whole numbers relate to multiplying a fraction by a whole number?
3. (Given two fractions with like denominators, each of which is less than $\frac{1}{2}$) Before adding these two fractions, can you predict whether the sum will be greater than or less than 1? How do you know?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students develop understanding of fractions as numbers and represent and solve problems involving multiplication and division.
3. This expectation connects to other ideas in Grade 4: (a) using decimal notation for fractions and comparing decimal fractions, (b) using the four operations with whole numbers to solve problems, (c) solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit, and (d) representing and interpreting data.
4. In Grade 5, students use equivalent fractions as a strategy to add and subtract fractions with unlike denominators and apply and extend previous understandings of multiplication to decimals.

COLORADO
Department of Education

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.NF.C. Number & Operations—Fractions: Use decimal notation for fractions, and compare decimal fractions.

Evidence Outcomes

Students Can:

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.) *For example, express* $\frac{3}{10}$ *as* $\frac{30}{100}$*, and add* $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$. (CCSS: 4.NF.C.5)
6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite* 0.62 *as* $\frac{62}{100}$*; describe a length as* 0.62 *meters; locate* 0.62 *on a number line diagram.* (CCSS: 4.NF.C.6)
7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model. (CCSS: 4.NF.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Approach adding, subtracting, and comparing problems with fractions and decimal fractions by reasoning about their values before or instead of applying an algorithm. (MP1)
2. Draw fraction models to reason about and compute with decimal fractions. (MP5)
3. Make use of the structure of place value to express and compare decimal numbers in tenths and hundredths. (MP7)

Inquiry Questions:

1. How does a fraction with a denominator of 10 or 100 relate to its decimal quantity?
2. How can visual models help to compare two decimal quantities?
3. How is locating a decimal on a number line similar to locating a fraction on a number line?

Coherence Connections:

1. This expectation represents major work of the grade.
2. This expectation connects to several ideas in Grade 4: (a) extending understanding of fraction equivalence and ordering, (b) building fractions from unit fractions, and (c) solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
3. In Grade 5, students understand the decimal place value system and use it with the four operations.

COLORADO
Department of Education

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.OA.A. Operations & Algebraic Thinking: Use the four operations with whole numbers to solve problems.

Evidence Outcomes

Students Can:

1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. (CCSS: 4.OA.A.1)
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See Appendix, Table 2) (CCSS: 4.OA.A.2)
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of multi-step word problems by understanding the relationships between known and unknown quantities. (MP1)
2. Reason quantitatively with word problems by considering the units involved and how the quantities they describe increase or decrease with addition and subtraction or scale with multiplication and division. (MP2)
3. Use mathematics to model real-world problems requiring operations with whole numbers and contextually interpret remainders when they arise. (MP4)
4. Look for structures of commutativity and inverses of operations in solving whole number problems with the four operations. (MP7)

Inquiry Questions:

1. What makes a multiplicative comparison different from an additive comparison?
2. How can you recognize whether a comparison is multiplicative or additive?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students represent and solve problems involving multiplication and division, apply properties of multiplication and the relationship between multiplication and division, solve problems involving the four operations, and identify and explain patterns in arithmetic.
3. This expectation connects to other ideas in Grade 4: (a) using place value understanding and properties of operations to perform multi-digit arithmetic, (b) extending understanding of fraction equivalence and ordering, (c) building fractions from unit fractions, and (d) solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4. In Grade 5, students apply and extend previous understandings of multiplication and division to multiply and divide fractions by fractions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

4.OA.B. Operations & Algebraic Thinking: Gain familiarity with factors and multiples.

Evidence Outcomes

Students Can:

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (CCSS: 4.OA.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason quantitatively to recognize that a number is a multiple of each of its factors. (MP2)
2. Use the relationship between factors and multiples for whole numbers. (MP7)
3. Look for, identify, and explain the regularities in determining whether a given number is a multiple of a given one-digit number and in determining if a given number is prime or composite. (MP8)

Inquiry Questions:

1. How can you use arrays to explore and determine all of the factors of a given number?
2. How are multiples and factors helpful in solving problems related to fractional parts of a whole number, such as $\frac{3}{5}$ of 20?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 3, students multiply and divide within 100.
3. In Grade 6, students compute fluently with multi-digit numbers, find common factors and multiples, and extend previous understandings of arithmetic to algebraic expressions.

COLORADO
Department of Education

Prepared Graduates:

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

4.OA.C. Operations & Algebraic Thinking: Generate and analyze patterns.

Evidence Outcomes

Students Can:

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* (CCSS: 4.OA.C.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Explore and generate sequences of numbers or shapes that can be described mathematically. (Entrepreneurial Skills: Creativity/Innovation)
2. Notice when calculations are repeated and describe patterns in generalized, mathematical ways. (MP8)

Inquiry Questions:

1. If you were given a rule to add 4 to a starting number then to each number that follows, can you generate a sequence of odd numbers? How?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 3, students solve problems involving the four operations and identify and explain patterns in arithmetic.
3. In Grade 5, students analyze pairs of patterns created from two given rules and describe and graph the corresponding relationships.

COLORADO
Department of Education

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

4.MD.A. Measurement & Data: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Evidence Outcomes

Students Can:

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that* 1 *ft is* 12 *times as long as* 1 *in. Express the length of a* 4 *ft snake as* 48 *in. Generate a conversion table for feet and inches listing the number pairs* (1,12), (2,24), (3,36), … (CCSS: 4.MD.A.1)
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (CCSS: 4.MD.A.2)
3. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* (CCSS: 4.MD.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Define quantities in measurement problems with both their magnitude and unit. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of quantities, their units, and their relationships in problem solving situations. (MP2)
3. Model real-world problems involving area and perimeter with equations, diagrams, and formulas, and use them to solve problems. (MP4)
4. Generate and use conversion tables to aid in measurement conversions, and represent measurement quantities on scaled line diagrams. (MP5)

Inquiry Questions:

1. How can you use what you know about place value to convert between km, m and cm? Does this also work for measurement of time (s, m, h)? Why or why not?
2. How can visual models help to make sense of measurement problems and intervals of time?
3. How many liters of juice are needed to fill 35 cups of 225 ml each?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 3, students solve problems involving multiplication and division and solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
3. In Grade 4, this expectation connects to building fractions from unit fractions, using decimal notation for fractions, comparing decimal fractions, and using the four operations with whole numbers to solve problems.
4. In Grade 5, students apply and extend previous understandings of multiplication and division, convert like measurement units within a given measurement system, and relate volume to multiplication and to addition.

Colorado Academic Standards
Mathematics

COLORADO
Department of Education

Prepared Graduates:

MP5. Use appropriate tools strategically.

Grade Level Expectation:

4.MD.B. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

4. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* (CCSS: 4.MD.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Read and represent measurements recorded on line plots. (Professional Skills: Information Literacy)
2. Use a line plot to represent measurement data and to calculate measurement sums and differences. (MP5)

Inquiry Questions:

1. Why is it helpful to organize data in line plots?
2. When might you see fractions in real-world data?
3. Why is it important to establish the whole when plotting fractions on a line plot?
4. How do labels help the reader determine the size of the numbers represented in a line plot?

Coherence Connections:

1. This GLE supports the major work of the grade.
2. In Grade 3, students represent and interpret data using picture graphs and scaled bar graphs.
3. In Grade 4, this expectation connects with building fractions from unit fractions.
4. In Grade 5, students represent and interpret data with line plots and solve problems using fractional measurements and all four operations.

COLORADO Department of Education

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

Grade Level Expectation:

4.MD.C. Measurement & Data: Geometric measurement: Understand concepts of angle and measure angles.

Evidence Outcomes

Students Can:

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: (CCSS: 4.MD.C.5)
   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. (CCSS: 4.MD.C.5.a)
   b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. (CCSS: 4.MD.C.5.b)
6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (CCSS: 4.MD.C.6)
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (CCSS: 4.MD.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze and measure the size of angles in real-world and mathematical problems. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason abstractly and quantitatively about angles and angular measurement. (MP2)

Inquiry Questions:

1. How is measuring angles with a protractor similar to measuring line segments with a ruler?
2. We can describe the fraction $\frac{3}{100}$ as $\frac{1}{100} + \frac{1}{100} + \frac{1}{100}$. How does this apply to the measurement of angles, such as an angle of 3 degrees?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 4, this expectation connects with drawing and identifying lines and angles, classifying shapes by properties of their lines and angles, and with understanding a fraction as a sum of unit fractions.
3. In Grade 7, students solve real-world and mathematical problems involving angle measure, area, surface area, and volume.

COLORADO
Department of Education

Prepared Graduates:

MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.G.A. Geometry: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Evidence Outcomes

Students Can:

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. (CCSS: 4.G.A.1)
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (CCSS: 4.G.A.2)
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. (CCSS: 4.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make observations and draw conclusions about the classification of two-dimensional figures based on the presence or absence of specified attributes. (Entrepreneurial Skills: Inquiry/Analysis)
2. Use appropriate tools strategically to draw lines (parallel, perpendicular, lines of symmetry), line segments, rays, and angles (right, acute, obtuse). (MP5)
3. Identify ways in which a shape is structured such that it displays line symmetry. (MP7)

Inquiry Questions:

1. Where do you see parallel lines, perpendicular lines, or lines of symmetry in the real world?
2. What kind of angle can you find most often in the real world: right, acute, or obtuse? Why do you think that is the case?
3. What kinds of shapes have many lines of symmetry and what kinds of shapes have no lines of symmetry?
4. In what ways might the lines of symmetry for a shape be related to dividing the shape into fractional parts?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students create composite shapes, recognize and draw shapes having specified attributes, and understand that shapes with shared attributes can define a larger category.
3. In Grade 4, this expectation connects with understanding concepts of angle and measuring angles.
4. In Grade 5, students classify two-dimensional figures into categories based on their properties.