

★  
Colorado  
Academic Standards

# Mathematics



**COLORADO**  
Department of Education

ALL STUDENTS • ALL STANDARDS

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## Purpose of Mathematics

“Pure mathematics is, in its way, the poetry of logical ideas.”

~Albert Einstein, *Obituary for Emmy Noether* (1935)

“Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics.”

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

**1. To Expand Professional Opportunity.** Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21<sup>st</sup>-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

**2. Understand and Critique the World.** A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

**3. Experience Wonder, Joy, and Beauty.** Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

## References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

## Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards:

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

## Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

### **1. Number and Quantity**

From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

### **2. Algebra and Functions**

Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

### **3. Data Analysis, Statistics, and Probability**

From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

### **4. Geometry**

Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

## Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.



### Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP7. Look for and make use of structure.

### Grade Level Expectation:

5.NBT.A. Number & Operations in Base Ten: Understand the place value system.

### Evidence Outcomes

#### *Students Can:*

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and  $\frac{1}{10}$  of what it represents in the place to its left. (CCSS: 5.NBT.A.1)
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. (CCSS: 5.NBT.A.2)
3. Read, write, and compare decimals to thousandths. (CCSS: 5.NBT.A.3)
  - a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g.,  $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times \frac{1}{10} + 9 \times \frac{1}{100} + 2 \times \frac{1}{1000}$ . (CCSS: 5.NBT.A.3.a)
  - b. Compare two decimals to thousandths based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons. (CCSS: 5.NBT.A.3.b)
4. Use place value understanding to round decimals to any place. (CCSS: 5.NBT.A.4)

### Academic Context and Connections

#### *Colorado Essential Skills and Mathematical Practices:*

1. Persist in making sense of how fractions can represent decimal place values. (Personal Skills: Perseverance/Resilience)
2. Abstract place value reasoning with whole numbers to decimal numbers. (MP2)
3. See the structure of place value as not just a making of tens with greater place values, but a making of tenths with lesser place values. (MP7)

#### *Inquiry Questions:*

1. How can you show visually the relationships between 25, 2.5 and 0.25? How can you show these relationships with equations?
2. Can all decimals be written as fractions? Why or why not?

#### *Coherence Connections:*

1. This expectation represents major work of the grade.
2. In Grade 4, students generalize place value understanding for multi-digit whole numbers, use decimal notation for fractions, and compare decimal fractions.
3. In Grade 5, this expectation connects with performing operations with multi-digit whole numbers and operations with decimals to hundredths.
4. In Grade 6, students apply and extend previous understandings of arithmetic to algebraic expressions and develop fluency with decimal operations.

### Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

MP7. Look for and make use of structure.

### Grade Level Expectation:

5.NBT.B. Number & Operations in Base Ten: Perform operations with multi-digit whole numbers and with decimals to hundredths.

### Evidence Outcomes

#### *Students Can:*

5. Fluently multiply multi-digit whole numbers using the standard algorithm. (CCSS: 5.NBT.B.5)
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 5.NBT.B.6)
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (CCSS: 5.NBT.B.7)

### Academic Context and Connections

#### *Colorado Essential Skills and Mathematical Practices:*

1. Defend calculations with explanations based on properties of operations, equations, drawings, arrays, and other models. (MP3)
2. Use models and drawings to represent and compute with whole numbers and decimals, illustrating an understanding of place value. (MP5)
3. Use the structure of place value to organize computation with whole numbers and decimals. (MP7)

#### *Inquiry Questions:*

1. We sometimes use arrays and area models to model multiplication and division of whole numbers. Do these models work for decimal fractions, too? Why or why not?
2. How is computation with decimal fractions similar to and different from computation with whole numbers?

#### *Coherence Connections:*

1. This expectation represents major work of the grade.
2. In Grade 4, students use place value understanding and properties of operations to perform multi-digit arithmetic.
3. This expectation connects with other ideas in Grade 5: (a) understanding the place value system for decimals, (b) using equivalent fractions as a strategy, (c) applying and extending previous understandings of multiplication and division, and (d) converting like measurement units within a given measurement system.
4. In Grade 6, students compute fluently with multi-digit numbers and find common factors and multiples.



### Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP6. Attend to precision.

MP7. Look for and make use of structure.

### Grade Level Expectation:

5.NF.A. Number & Operations—Fractions: Use equivalent fractions as a strategy to add and subtract fractions.

### Evidence Outcomes

#### *Students Can:*

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,*  $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$ . *(In general,*  $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ .*)* (CCSS: 5.NF.A.1)
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result*  $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$  *by observing that*  $\frac{3}{7} < \frac{1}{2}$ . (CCSS: 5.NF.A.2)

### Academic Context and Connections

#### *Colorado Essential Skills and Mathematical Practices:*

1. Construct viable arguments about the addition and subtraction of fractions with reasoning rooted in the need for like-sized parts. (MP3)
2. Assess the reasonableness of fraction calculations by estimating results using benchmark fractions and number sense. (MP6)
3. Look for structure in the multiplicative relationship between unlike denominators when creating equivalent fractions. (MP7)

#### *Inquiry Questions:*

1. It is useful to round decimals when estimating sums and differences of decimal numbers. What would “rounding fractions” look like when estimating sums and differences of fractions?
2. Why don’t we add or subtract the denominators when we are working with fractions?

#### *Coherence Connections:*

1. This expectation represents major work of the grade.
2. In Grade 4, students add and subtract fractions and mixed numbers with like denominators, recognize and generate equivalent fractions, and compare fractions with different numerators and denominators.
3. In Grade 5, this expectation connects with multi-digit whole number operations, operations with decimals to hundredths, and representing and interpreting data.
4. In Grade 6, students reason about and solve one-variable equations and inequalities, and in Grade 7, apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

### Prepared Graduates:

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

### Grade Level Expectation:

5.NF.B. Number & Operations—Fractions: Apply and extend previous understandings of multiplication and division.

### Evidence Outcomes

#### Students Can:

3. Interpret a fraction as division of the numerator by the denominator ( $\frac{a}{b} = a \div b$ ). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret  $\frac{3}{4}$  as the result of dividing 3 by 4, noting that  $\frac{3}{4}$  multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size  $\frac{3}{4}$ . If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* (CCSS: 5.NF.B.3)
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. (CCSS: 5.NF.B.4)
  - a. Interpret the product  $\frac{a}{b} \times q$  as a parts of a partition of  $q$  into  $b$  equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ . *For example, use a visual fraction model to show  $\frac{2}{3} \times 4 = \frac{8}{3}$  and create a story context for this equation. Do the same with  $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$ . (In general,  $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ .)* (CCSS: 5.NF.B.4.a)
  - b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (CCSS: 5.NF.B.4.b)
5. Interpret multiplication as scaling (resizing), by: (CCSS: 5.NF.B.5)
  - a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. (CCSS: 5.NF.B.5.a)
  - b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence  $\frac{a}{b} = \frac{n \times a}{n \times b}$  to the effect of multiplying  $\frac{a}{b}$  by 1. (CCSS: 5.NF.B.5.b)
6. Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (CCSS: 5.NF.B.6)
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.) (CCSS: 5.NF.B.7)
  - a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for  $\frac{1}{3} \div 4$ , and use a visual fraction model to show the quotient. Use the*



- relationship between multiplication and division to explain that  $\frac{1}{3} \div 4 = \frac{1}{12}$  because  $\frac{1}{12} \times 4 = \frac{1}{3}$ . (CCSS: 5.NF.B.7.a)
- b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for  $4 \div \frac{1}{5}$  and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $4 \div \frac{1}{5} = 20$  because  $20 \times \frac{1}{5} = 4$ . (CCSS: 5.NF.B.7.b)
- c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share  $\frac{1}{2}$  lb of chocolate equally? How many  $\frac{1}{3}$ -cup servings are in 2 cups of raisins? (CCSS: 5.NF.B.7.c)

### Academic Context and Connections

#### *Colorado Essential Skills and Mathematical Practices:*

1. Solve problems requiring calculations that scale whole numbers and fractions. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Use fraction models and arrays to interpret and explain fraction calculations. (MP5)
3. Attend carefully to the underlying unit quantities when solving problems involving multiplication and division of fractions. (MP6)
4. Contrast previous understandings of multiplication modeled as equal groups to multiplication as scaling, which is necessary to understand multiplying a fraction or whole number by a fraction, and how the operation of multiplication does not always result in a product larger than both factors. (MP7)

#### *Inquiry Questions:*

1. How can you rewrite the fraction  $\frac{5}{3}$  with an addition equation? How can you rewrite it with a multiplication equation? How does it make sense that both equations are accurate?
2. If we can describe the product of  $5 \times 3$  as “three times as big as 5,” what does that tell us about the product of  $5 \times \frac{1}{2}$ ? What about  $\frac{1}{5} \times \frac{1}{2}$ ?

#### *Coherence Connections:*

1. This expectation represents major work of the grade.
2. In previous grades, students base understanding of multiplication on its connection to addition, groups of equivalent objects, and area models. In Grade 4, students add and subtract fractions and mixed numbers with like denominators, recognize and generate equivalent fractions, and compare fractions with different numerators and denominators.
3. This expectation connects with several others in Grade 5: (a) performing operations with multi-digit whole numbers and with decimals to hundredths, (b) writing and interpreting numerical expressions, and (c) representing and interpreting data.
4. In Grade 6, students (a) understand ratio concepts and use ratio reasoning to solve problems, (b) apply and extend previous understandings of multiplication and division to divide fractions by fractions, (c) reason about and solve one-variable equations and inequalities, and (d) solve real-world and mathematical problems involving area, surface area, and volume.



### Prepared Graduates:

MP7. Look for and make use of structure.

### Grade Level Expectation:

5.OA.A. Operations & Algebraic Thinking: Write and interpret numerical expressions.

### Evidence Outcomes

#### *Students Can:*

1. Use grouping symbols (parentheses, brackets, or braces) in numerical expressions, and evaluate expressions with these symbols. (CCSS: 5.OA.A.1)
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation “add 8 and 7, then multiply by 2” as  $2 \times (8 + 7)$ . Recognize that  $3 \times (18932 + 921)$  is three times as large as  $18932 + 921$ , without having to calculate the indicated sum or product.* (CCSS: 5.OA.A.2)

### Academic Context and Connections

#### *Colorado Essential Skills and Mathematical Practices:*

1. Write expressions that represent mathematical relationships between quantities. (Entrepreneurial Skills: Literacy/Writing)
2. Look for structures and notation that make the order of operations clear when reading and writing mathematical expressions. (MP7)

#### *Inquiry Questions:*

1. How can you describe the relationship between the value of  $5 \times (24562 + 951)$  and  $24562 + 951$  without making any calculations?
2. Suppose we use the letter  $a$  to represent a number. Can you determine the relationship between  $4 \times a$  and  $a$  without knowing the specific number  $a$  represents?

#### *Coherence Connections:*

1. This expectation is in addition to the major work of the grade.
2. In Grade 5, this expectation connects with applying and extending previous understandings of multiplication and division.
3. In Grade 6, students compute fluently with multi-digit numbers, find common factors and multiples, and apply and extend previous understandings of arithmetic to algebraic expressions. In middle and high school, students develop fluency with algebraic expressions as mathematical objects that can be used in more complex mathematical operations.



### Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP8. Look for and express regularity in repeated reasoning.

### Grade Level Expectation:

5.OA.B. Operations & Algebraic Thinking: Analyze patterns and relationships.

### Evidence Outcomes

#### *Students Can:*

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* (CCSS: 5.OA.B.3)

### Academic Context and Connections

#### *Colorado Essential Skills and Mathematical Practices:*

1. Analyze and compare patterns. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason quantitatively with patterns by relating sequences of numbers with the rule that generated them. (MP3)
3. Look for repeated reasoning both within individual patterns and in mathematical relationships between pairs of patterns. (MP8)

#### *Inquiry Questions:*

1. When you graph the corresponding terms formed by two numerical rules, how are the rules reflected in the graph?
2. How does the relationship between two patterns generated by rules relate to the rules themselves?

#### *Coherence Connections:*

1. This expectation is in addition to major work of the grade.
2. In Grade 4, students generate and analyze number or shape patterns and generalize about them.
3. In Grade 6, students understand ratio concepts, use ratio reasoning to solve problems, extend previous understandings of arithmetic to algebraic expressions, and represent and analyze quantitative relationships between dependent and independent variables.



### Prepared Graduates:

MP6. Attend to precision.

### Grade Level Expectation:

5.MD.A. Measurement & Data: Convert like measurement units within a given measurement system.

### Evidence Outcomes

#### *Students Can:*

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems. (CCSS: 5.MD.A.1)

### Academic Context and Connections

#### *Colorado Essential Skills and Mathematical Practices:*

1. Convert measurements to solve real-world problems. (Professional Skills: Information Literacy)
2. Use appropriate precision when converting measurements based on a problem's context. (MP6)

#### *Inquiry Questions:*

1. What is happening mathematically when we convert from centimeters to meters? What about when we convert from meters to centimeters?
2. How can you use fractions to change 53 kilograms to grams? How can you use decimals to do this conversion?

#### *Coherence Connections:*

1. This expectation supports the major work of the grade.
2. In Grade 4, students solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
3. In Grade 5, this expectation connects with performing operations with multi-digit whole numbers and with decimals to hundredths.



### Prepared Graduates:

MP5. Use appropriate tools strategically.

### Grade Level Expectation:

5.MD.B. Measurement & Data: Represent and interpret data.

### Evidence Outcomes

#### *Students Can:*

2. Make a line plot to display a data set of measurements in fractions of a unit ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* (CCSS: 5.MD.B.2)

### Academic Context and Connections

#### *Colorado Essential Skills and Mathematical Practices:*

1. Display fractional measurement data in line plots. (Professional Skills: Information Literacy)
2. Participate in discussions of measurement data using information presented in line plots. (Civic/Interpersonal Skills: Literacy/Oral Expression and Listening)
3. Strategically determine the scale of line plots to represent fractional measurements. (MP5)

#### *Inquiry Questions:*

1. (Given a data set of fractional measurements with unlike denominators) What will you consider in deciding how to label the tick marks on the line for your line plot?

#### *Coherence Connections:*

1. This expectation supports the major work of the grade.
2. In Grade 4, students represent and interpret data.
3. In Grade 5, this expectation connects with using equivalent fractions and applying and extending previous understandings of multiplication and division.
4. In Grade 6, students develop understanding of statistical variability and summarize and describe distributions.



### Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP5. Use appropriate tools strategically.

MP7. Look for and make use of structure.

### Grade Level Expectation:

5.MD.C. Measurement & Data: Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.

### Evidence Outcomes

#### *Students Can:*

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. (CCSS: 5.MD.C.3)
  - a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume and can be used to measure volume. (CCSS: 5.MD.C.3.a)
  - b. A solid figure which can be packed without gaps or overlaps using  $n$  unit cubes is said to have a volume of  $n$  cubic units. (CCSS: 5.MD.C.3.b)
4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. (CCSS: 5.MD.C.4)
5. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. (CCSS: 5.MD.C.5)
  - a. Model the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. (CCSS: 5.MD.C.5.a)
  - b. Apply the formulas  $V = l \times w \times h$  and  $V = b \times h$  for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. (CCSS: 5.MD.C.5.b)

- c. Use the additive nature of volume to find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems. (CCSS: 5.MD.C.5.c)

### Academic Context and Connections

#### *Colorado Essential Skills and Mathematical Practices:*

1. Solve real-world problems involving volume. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make connections between the values being multiplied in a volume formula, the concept of cubic units, and the context within which volume is being calculated. (MP2)
3. Use unit cubes as a tool for finding or estimating volume and compare those results with those obtained with formulas. (MP5)
4. Extend the structure of two-dimensional space and the relationship between arrays and area to three-dimensional space and the relationship between layers of cubes and volume. (MP7)

#### *Inquiry Questions:*

1. How are volume and area related in a solid figure?
2. Why is multiplication used when computing the volume of a solid figure, instead of another operation?





*Coherence Connections:*

1. This expectation represents major work of the grade.
2. In previous grades, students connect area to the operation of multiplication and understand how to represent area problems as multiplication equations.
3. In Grade 6, students solve real-world and mathematical problems involving area of right rectangular prisms with fractional side lengths, using fractional cubic units.



### Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP5. Use appropriate tools strategically.

### Grade Level Expectation:

5.G.A. Geometry: Graph points on the coordinate plane to solve real-world and mathematical problems.

### Evidence Outcomes

#### *Students Can:*

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g.,  $x$ -axis and  $x$ -coordinate,  $y$ -axis and  $y$ -coordinate). (CCSS: 5.G.A.1)
2. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (CCSS: 5.G.A.2)

### Academic Context and Connections

#### *Colorado Essential Skills and Mathematical Practices:*

1. Use the first quadrant of the coordinate plane to represent real-world and mathematical problems. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Analyze and use information presented visually in a coordinate plane. (Entrepreneurial Skills: Literacy/Reading)
3. Reason quantitatively about a problem by abstracting and representing the situation in the first quadrant of the coordinate plane. (MP2)
4. Use the first quadrant of the coordinate plane as a tool to represent, analyze, and solve problems. (MP5)

#### *Inquiry Questions:*

1. What are things in the real world that are designed like a coordinate plane or that use a coordinate system?
2. Why are the axes of the coordinate plane made to form right angles instead of acute and obtuse angles?

#### *Coherence Connections:*

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students use number lines to represent whole and fractional number distances from zero.
3. In Grade 6, students extend the number line and the coordinate plane to include negative numbers and solve real-world and mathematical problems by graphing points in all four quadrants.



### Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

MP7. Look for and make use of structure.

### Grade Level Expectation:

5.G.B. Geometry: Classify two-dimensional figures into categories based on their properties.

#### Evidence Outcomes

##### *Students Can:*

3. Explain that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* (CCSS: 5.G.B.3)
4. Classify two-dimensional figures in a hierarchy based on properties. (CCSS: 5.G.B.4)

#### Academic Context and Connections

##### *Colorado Essential Skills and Mathematical Practices:*

1. Observe and analyze attributes of two-dimensional figures to classify them. (Entrepreneurial Skills: Inquiry/Analysis)
2. Critique the reasoning of others' classifications of two-dimensional shapes. (MP3)
3. Strategically use measurement tools to help improve the classification of shapes. (MP5)
4. Look for and use attributes of two-dimensional shapes to classify the shapes in a hierarchy of figures. (MP7)

##### *Inquiry Questions:*

1. How can you use the words “always,” “sometimes,” and “never” to develop a classification of two-dimensional figures?

##### *Coherence Connections:*

1. This expectation is in addition to the major work of the grade.
2. In Grade 4, students draw and identify lines and angles and classify shapes by properties of their lines and angles.